Appendix A
Stability Analysis of the ErrorS Algorithm

We model the ErrorS algorithm using the fluid model of a single bottleneck network in order to study the stability of the ErrorS algorithm as a function of the values of α, β and μ. Abstracting from packet granularity, neglecting queuing delay, and assuming a continuous control loop, the ErrorS system is characterized by the following equations:

$$\dot{y}(t) = \frac{F(t-d)}{d} \tag{1}$$

$$F(t) = \alpha \cdot (C - y(t)) - \frac{\mu}{d} \cdot \int_0^t \frac{\beta}{\alpha \cdot d} \cdot q(t)dt - \frac{\beta}{d} \cdot q(t) \tag{2}$$

$$\dot{q}(t) = y(t) - C_{real} \tag{3}$$

$$C = C_{real} + \epsilon \tag{4}$$

where $y(t)$ represents the aggregate incoming bandwidth, C is the capacity value configured at the router, C_{real} is the actual bottleneck link capacity, ϵ is the router capacity estimate error, $q(t)$ represents queue length, and d is the system (average) RTT. α, β and μ are the tunable constants of the algorithm. Now consider the following nomenclature simplification:

$$K_1 = \frac{\alpha}{d}; K_2 = \frac{\beta}{d^2}; K_3 = \frac{\mu \cdot \beta}{\alpha \cdot d^3} \tag{5}$$

which allows us to rewrite Eq. 1 and Eq. 2 as:

$$\begin{aligned} \dot{y}(t) = \quad & K_1 \cdot (C - y(t-d)) - \\ & K_3 \cdot \textstyle\int_0^t q(t-d) \cdot u(t-d)dt - \\ & K_2 \cdot q(t-d) \end{aligned} \tag{6}$$

where $u(t-d)$ represents a delayed Heavyside step function. Applying the Laplace transformation we finally get:

$$s \cdot Y(s) = e^{-sd} \left(K_1 \cdot (C - Y(s)) - K_3 \cdot \frac{Q(s)}{s} - K_2 \cdot Q(s) \right) \tag{7}$$

which is equivalent to the negative feedback control loop represented in Fig. 1. The open-loop transfer function $G(s)$ of the the system is given by:

$$G(s) = e^{-sd} \cdot \frac{K_1 \cdot s^2 + K_2 \cdot s + K_3}{s^3} \tag{8}$$

In order to ensure system stability we have to guarantee that the phase (or gain) margin of the open-loop transfer function is positive. To simplify the analysis consider the following condition:

$$\mu = \beta \tag{9}$$

which implies:

$$K_3 = \frac{K_2^2}{K_1} \tag{10}$$

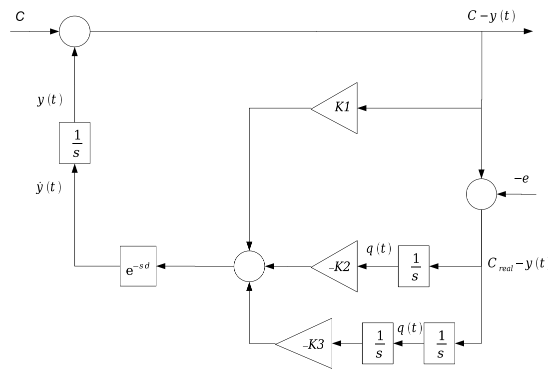

Fig. 1: The ErrorS control loop.

Using this condition the zeros w_z of $G(s)$ are:

$$w_z = \frac{K_2}{2 \cdot K_1} \cdot (1 \pm i\sqrt{3}) \tag{11}$$

Now we have to choose the cut-off frequency w_{ErrorS} of the system. For the sake of tractability we consider the cut-off frequency as a multiple of the real part of the frequency of the w_z:

$$w_{ErrorS} = n \cdot \Re\{w_z\} = n \cdot \frac{K_2}{2 \cdot K_1} \tag{12}$$

Applying the condition of unitary gain to the cut-off frequency:

$$|G(jw_{ErrorS})| = 1 \tag{13}$$

we get:

$$\beta = \frac{\sqrt{1 - \left(\frac{n}{2}\right)^2 + \left(\frac{n}{2}\right)^4}}{\left(\frac{n}{2}\right)^3} \cdot \alpha^2 \tag{14}$$

System stability is ensured if the open-loop response phase does not exceed $-\pi$ at the cut-off frequency:

$$\angle G(jw_{ErrorS}) > -\pi \tag{15}$$

which results in:

$$\frac{\beta}{\alpha} \leq \frac{2}{n} \cdot \left[\arctan\left(\frac{n}{2 - \left(\frac{n}{\sqrt{2}}\right)^2} \right) - \frac{\pi}{2} \right] \tag{16}$$

The final step of the stability analysis consists in selecting the actual cut-off frequency of the open-loop response by choosing a value for n. We choose n so that the error is surpressed as fast as possible, and the importance given to the erroneous estimation is as low as possible. Considering that the speed of the error suppression is controlled by the parameter μ ($\mu = \beta$), and that α is the weight given to the erroneous capacity estimation, we choose the value of n which maximizes the $\frac{\beta}{\alpha}$. In

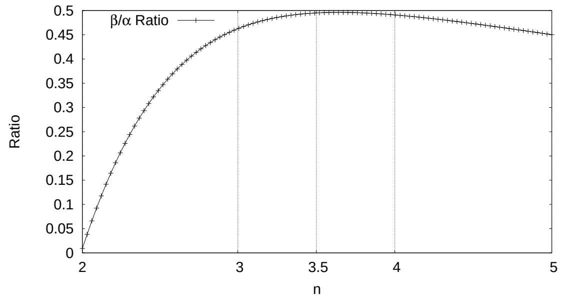

Fig. 2: The maximum value of β as a function of n.

Fig. 2 we plot this ratio as a function of n, where it is observable that a maximum is obtained for $n \approx 3.5$. For $n = 3.5$ we get a system cut-off frequency w_{ErrorS}:

$$w_{ErrorS} = \frac{3.5}{2} \cdot \frac{K_2}{K_1} = 1.75 \cdot \frac{\beta}{\alpha \cdot d} \qquad (17)$$

Applying $n = 3.5$ in Eq. 14 and Eq. 16 we can extract the final set of stability restrictions for the parameters α, β and μ:

$$\beta = 0.5047 \cdot \alpha^2; \qquad \frac{\beta}{\alpha} \leq 0.4955; \qquad \mu = \beta \qquad (18)$$

which, simplifying, becomes:

$$\beta = 0.5047 \cdot \alpha^2; \qquad \alpha \leq 0.9818; \qquad \mu = \beta \qquad (19)$$

In order to reduce amplitude oscillation and increase system robustness to fluctuations, we add a phase margin. Based on simulation results we recommend the utilization of the following values:

$$\beta = 0.1817; \qquad \alpha = 0.6; \qquad \mu = 0.1817 \qquad (20)$$

for which the ErrorS algorithm is stable independently of link capacity, delay or number of sources.

Appendix B
Recommended Parameter Values for the Blind and the ErrorS Algorithms

	Blind	ErrorS
α	0.4	0.6
β	0.226	0.1817
μ	-	0.1817
ρ	0.22	0.15
Q_χ	$0.541 \cdot Q_{max}$	$0.444 \cdot Q_{max}$
τ	0.225	0.37

TABLE 1: Recommended parameter values for the Blind and the ErrorS algorithms

We adopt the recommended values of α, β in Blind from [11], as the Blind algorithm maintains the control dynamics of the XCP algorithm. Similarly to the ErrorS algorithms, we fix $\alpha = 0.6$ - allowing a reasonable stability margin - and use the relations found in Appendix A, which guarantee stability, to dimension β, μ. The recommended parameter values are shown in Table 1.